M13/5/MATHL/HP3/ENG/TZ0/SG

22137209

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 3 – SETS, RELATIONS AND GROUPS

Tuesday 21 May 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

2213-7209

3 pages

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 10]*

The binary operation $*$ is defined on $\mathbb{N}$ by $a * b = 1 + ab$.

Determine whether or not $*$

(a) is closed; *[2 marks]*

(b) is commutative; *[2 marks]*

(c) is associative; *[3 marks]*

(d) has an identity element. *[3 marks]*

2. *[Maximum mark: 16]*

Consider the set $S = \{1, 3, 5, 7, 9, 11, 13\}$ under the binary operation multiplication modulo 14 denoted by $\times_{14}$.

(a) Copy and complete the following Cayley table for this binary operation.

$\times_{14}$	**1**	**3**	**5**	**7**	**9**	**11**	**13**
1	1	3	5	7	9	11	13
3	3				13	5	11
5	5				3	13	9
7	7						
9	9	13	3				
11	11	5	13				
13	13	11	9				

[4 marks]

(b) Give one reason why $\{S, \times_{14}\}$ is not a group. *[1 mark]*

(c) Show that a new set G can be formed by removing one of the elements of S such that $\{G, \times_{14}\}$ is a group. *[5 marks]*

(This question continues on the following page)

(Question 2 continued)

(d) Determine the order of each element of $\{G, \times_{14}\}$. *[4 marks]*

(e) Find the proper subgroups of $\{G, \times_{14}\}$. *[2 marks]*

3. *[Maximum mark: 13]*

The function $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 2x+1 & \text{for } x \le 2 \\ x^2 - 2x + 5 & \text{for } x > 2 \, . \end{cases}$$

(a) (i) Sketch the graph of f.

(ii) By referring to your graph, show that f is a bijection. *[5 marks]*

(b) Find $f^{-1}(x)$. *[8 marks]*

4. *[Maximum mark: 13]*

The relation R is defined on $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ by aRb if and only if $a(a+1) \equiv b(b+1) (\text{mod} 5)$.

(a) Show that R is an equivalence relation. *[6 marks]*

(b) Show that the equivalence defining R can be written in the form

$$(a-b)(a+b+1) \equiv 0 (\text{mod} 5) \, .$$ *[3 marks]*

(c) Hence, or otherwise, determine the equivalence classes. *[4 marks]*

5. *[Maximum mark: 8]*

H and K are subgroups of a group G. By considering the four group axioms, prove that $H \cap K$ is also a subgroup of G.